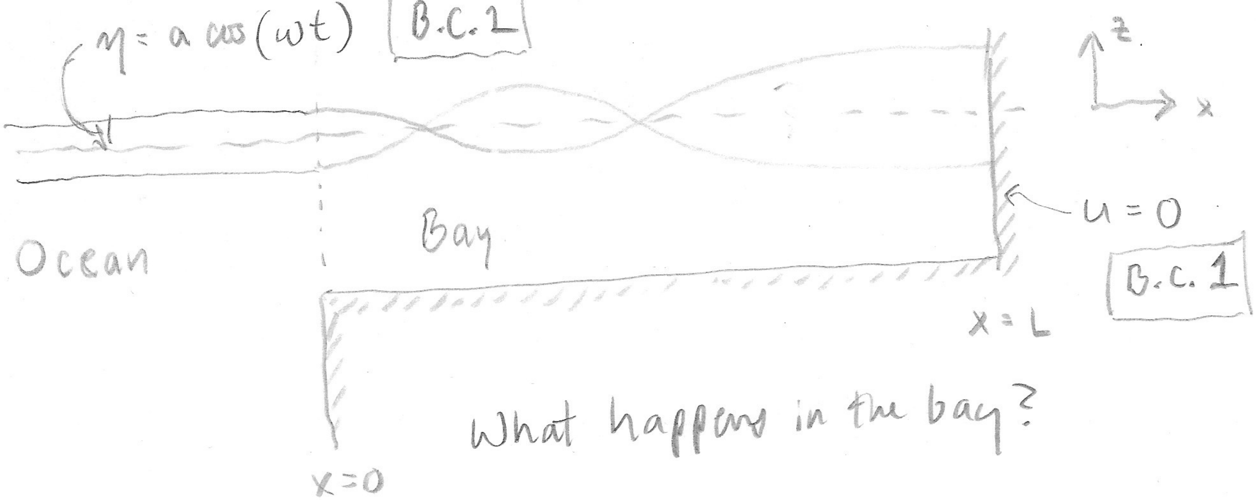
(5) $\boxed{\text{SW Tides w/ Friction in a Closed Bay: the "Quarter Wave Oscillator"}}$

7/26/2019 ①



What happens in the bay?

General solution:

$$\eta = \mathrm{Re}\{\underbrace{\alpha^+ \exp i(kx - \omega t)}_{\text{incident}} + \underbrace{\alpha^- \exp i(-kx-\omega t)}_{\text{reflected}}\}$$

$$k = \frac{\omega}{c}\sqrt{1 + i\frac{R}{\omega}}$$

- $\boxed{\text{B.C. 1}}$ $u = 0$ at $x = L$

$\boxed{\text{x mom}}$ $u_t + g\eta_x + Ru = 0$; $u = 0 \Rightarrow \boxed{\eta_x = 0}$

satisfy b.c. by making incident + reflected match at $x = L$

$$\Rightarrow \quad \alpha^+ = \alpha \exp i(-kL), \quad \alpha^- = \alpha \exp i(kL)$$

α = unknown complex const.

$$\Rightarrow \eta = \mathrm{Re}\{\alpha[\exp ik(x-L) + \exp -ik(x-L)]e^{-i\omega t}\}$$

and this can be simplified:

$$\eta = \mathrm{Re}\left\{ \alpha\left[\cos k(x-L) + i\cancel{\sin k(x-L)} + \cos k(x-L) - i\cancel{\sin k(x-L)} \right] e^{-i\omega t} \right\}$$

$$\Rightarrow \eta = \mathrm{Re}\left\{ 2\alpha \cos k(x-L)\, e^{-i\omega t} \right\}$$ satisfies $\eta_x = 0$ at $x = L$ ✓

- $\boxed{\text{B.C. 2}}$ $\eta = a\cos\omega t$ at $x = 0$

  satisfied by choosing $2\alpha \cos kL = a \Rightarrow 2\alpha = \dfrac{a}{\cos kL}$

$\Rightarrow$ exact solution

$$\boxed{\eta = \mathrm{Re}\left\{ \frac{a\cos k(x-L)}{\cos kL}\, e^{-i\omega t} \right\}} \quad (\ast)$$

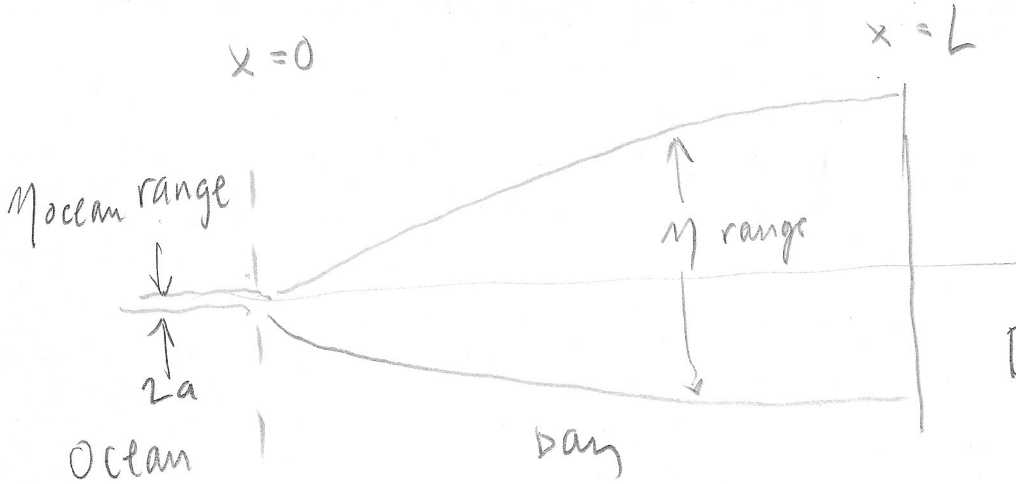
- can use inviscid solution (k = real) to understand resonance
- with friction: evaluate w/ python or matlab

Resonance (ignore friction):

For a bay of a certain length: $kL = \frac{\pi}{2} \rightarrow \cos(kL) = 0$ and η blows up. This is at the lowest resonant frequency. Since $kx = 2\pi$ for a full wavelength, $kL = \pi/2$ is a quarter wavelength with a node at the mouth. Call this $L_{1/4}$.

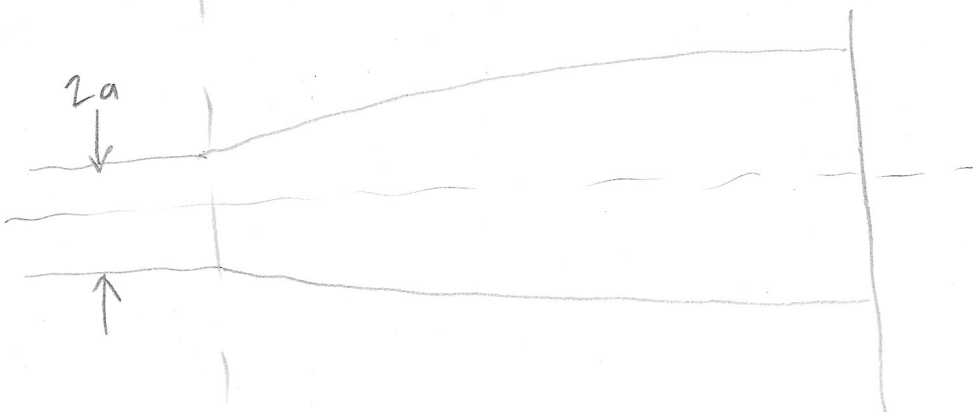
(Note $k = \frac{\omega}{c} = \frac{\omega}{\sqrt{gH}}$ so we can also cast this as a resonant frequency for any L, H: $\boxed{\omega_{1/4} = \frac{\pi}{2}\frac{\sqrt{gH}}{L}}$

⊛ Calculate $\omega_{1/4}$ for your system

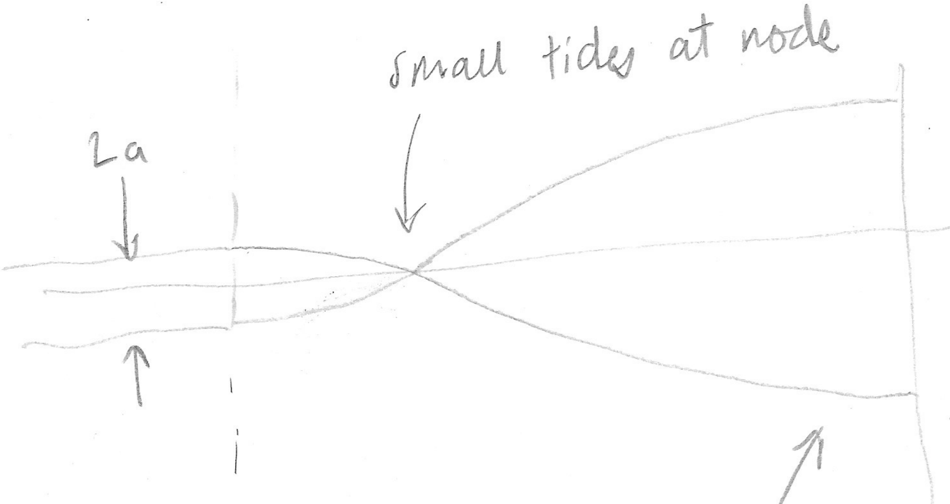


$\omega \sim \omega_{1/4}$

Biggest amplification



$\omega < \omega_{1/4}$



$\omega > \omega_{1/4}$

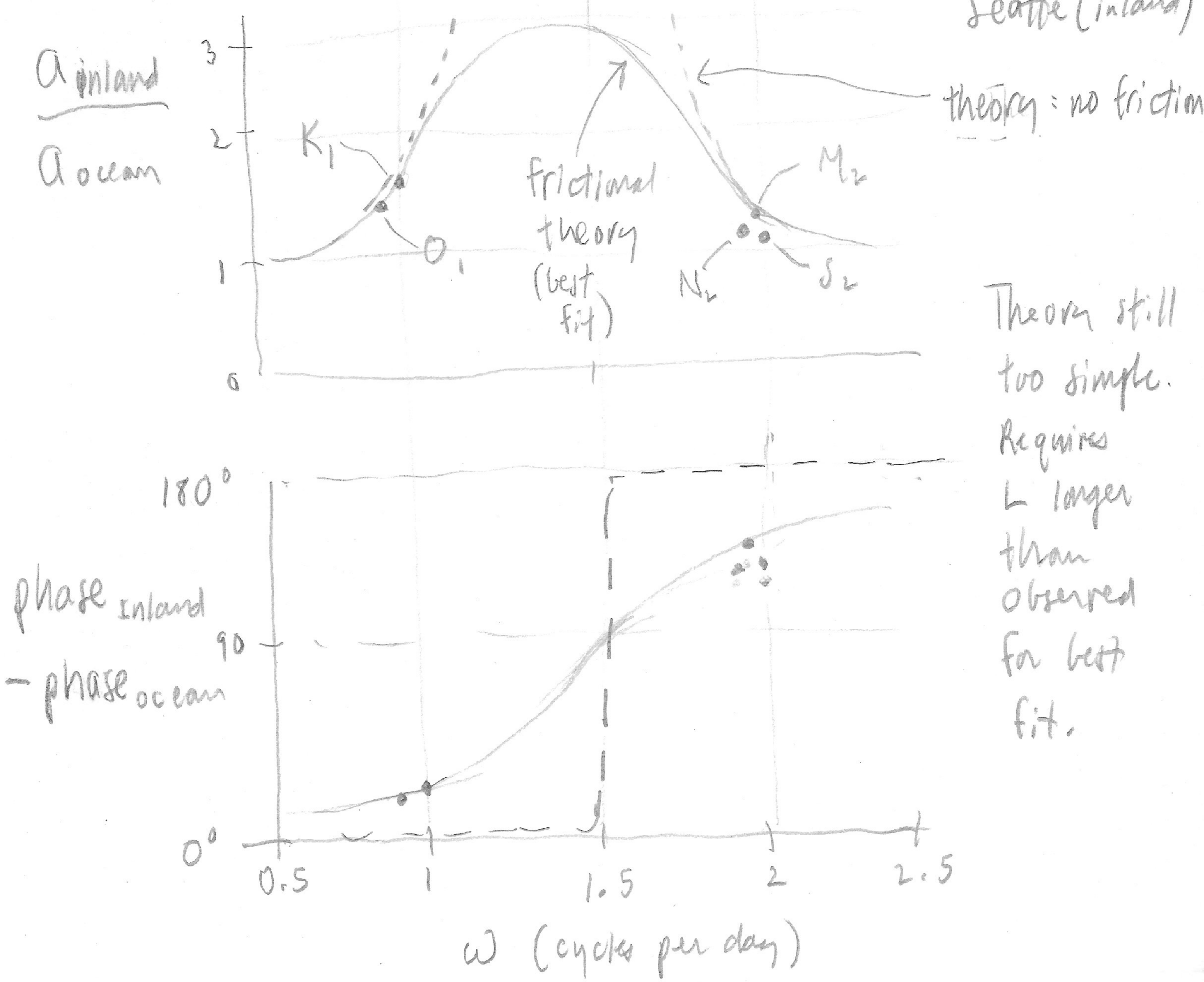
Inland response is 180° out of phase with ocean

(*) Q: at $x = L$ are tides ever smaller than in ocean?

(A) Draw Pattern of Velocity at 4 times for $\omega > \omega_{1/4}$ case

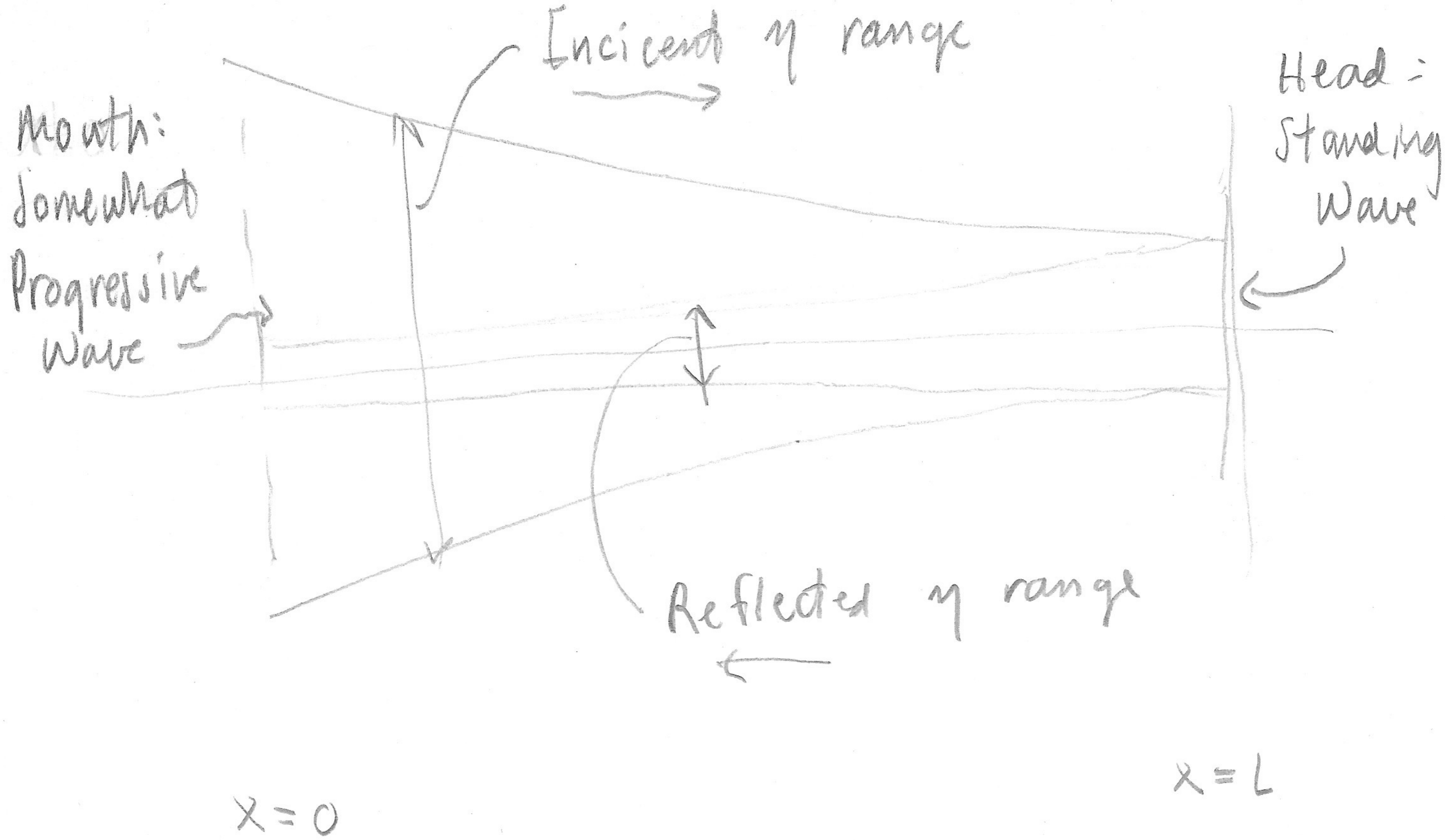
- Using the theory to understand the Salish Sea (See Sutherland et al. 2005 on syllabus).

- Need to include some friction ⇒ evaluate numerically (e.g. sw_tides.py on syllabus)
- Data from analysis of tides at La Push (ocean) and Seattle (inland)



Theory still too simple. Requires L longer than observed for best fit.

Note on the frictional solution:
it is the sum of wave that
decay in two directions



(A) Can you see progressive vs. standing
in η pattern of sw-tides.py plot?